Maxwell's Equations and Electromagnetic Waves

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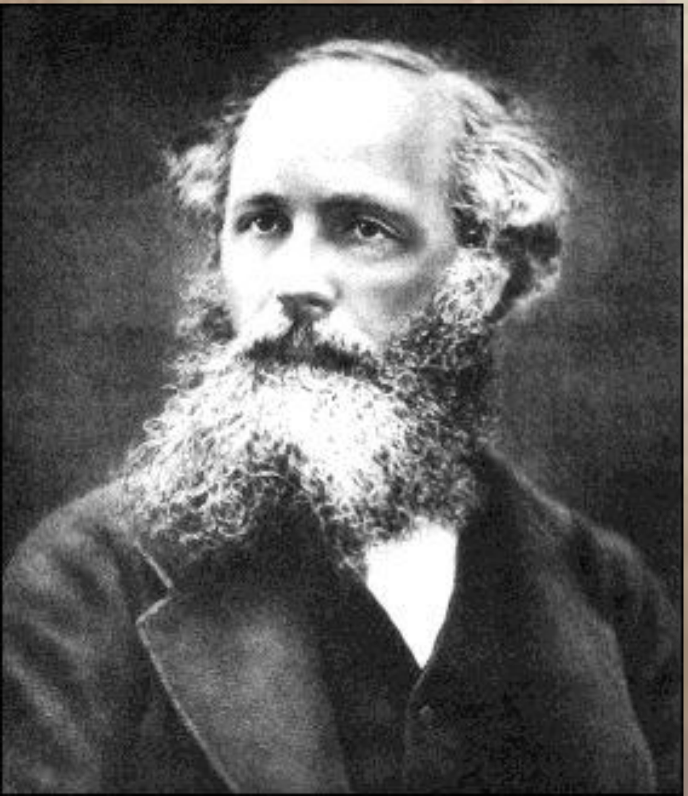
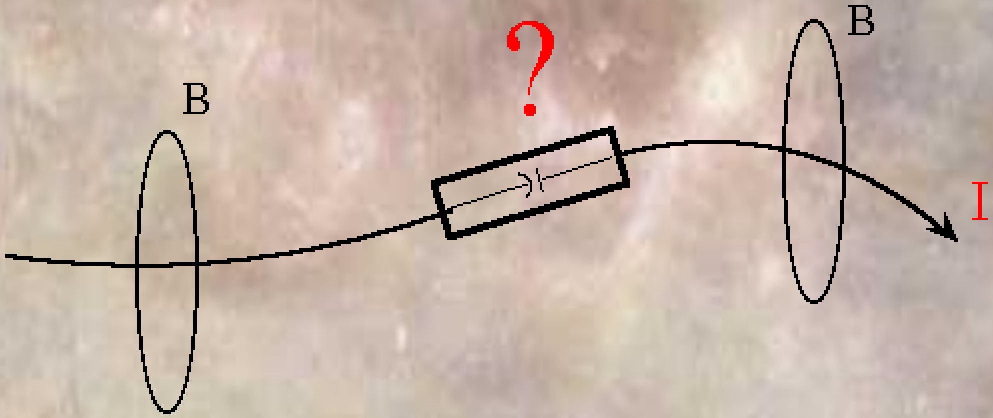
Setting the Stage - The Displacement Current

- Maxwell had a crucial "leap of insight"...



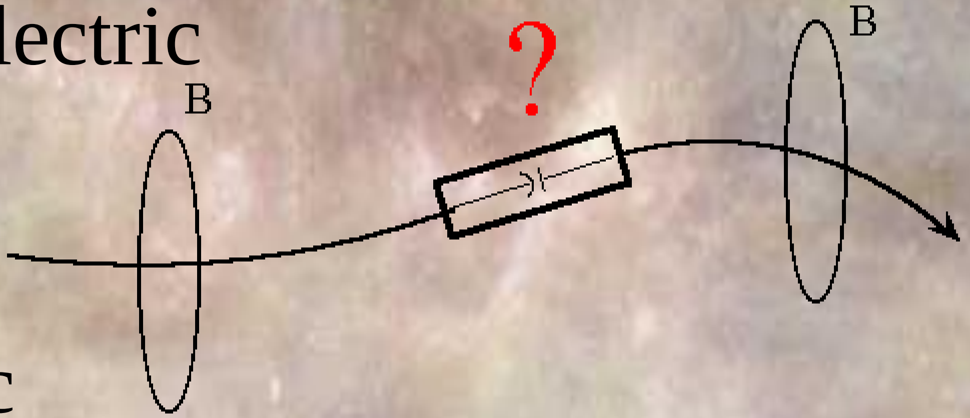


Will there still be a magnetic field around the capacitor?

A Beautiful Symmetry...

- A changing magnetic flux produces an Electric field

- A changing electric flux produces a Magnetic field



An extension of Ampere's Law...

- Maxwell reasoned that Ampere's Law would also apply to the displacement current.

$$\int Bdl = \mu_o (I + I_{displacement})$$

Clever application of Gauss' Law here!

$$\varepsilon_o \frac{d\phi_E}{dt} = \frac{dQ}{dt} = I_{displacement}$$

Maxwell's Equations (first glimpse)

- Faraday's Law:

$$\int E dl = -\frac{d}{dt}\int B_n dA$$

- Ampere's Law:

$$\int B dl = \mu_o I + \mu_o \varepsilon_o \frac{d}{dt}\int E_n dA$$

Maxwell's Equations – Integral Form

- Faraday's Law: $$\int E dl = -\frac{d}{dt}\int B_n dA$$
- Ampere's Law: $$\int B dl = \mu_o I + \mu_o \varepsilon_o \frac{d}{dt}\int E_n dA$$
- Gauss' Law $$\int E \cdot dA = \frac{Q}{\varepsilon_o}$$

$$\int B \cdot dA = 0$$

Maxwell's Equations – Differential Form

$$\int E \cdot dA = \int_V (\nabla \cdot E) dV = \frac{Q}{\varepsilon_o}$$

$$Q = \int_V \rho dV$$

$$\nabla \cdot E = \frac{\rho}{\varepsilon_o}$$

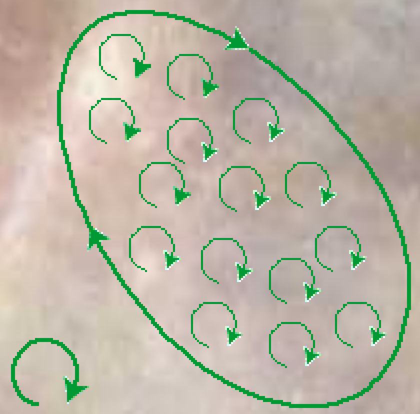
Gauss' Theorem – integral of a flux equals volume integral of divergence

Maxwell's Equations – Differential Form

$$\int E dl = -\frac{d}{dt}\int B_n dA$$

$$\int_\Gamma E \cdot dl = \int_\Sigma (\nabla \times E) dA = -\frac{d}{dt}\int B_n dA$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$



Stoke's Theorem: "Integral around the path equals flux of the curl"

Maxwell's Equations – Differential Form

$$\int B dl = \int_{\Sigma} (\nabla \times B) dA = \mu_o \varepsilon_o \frac{d}{dt} \int E_n dA$$

$$\nabla \times B = \mu_o \varepsilon_o \frac{\partial E}{\partial t}$$

Maxwell's Equations – Differential Form

$$\nabla \cdot E = \frac{\rho}{\varepsilon_o}$$

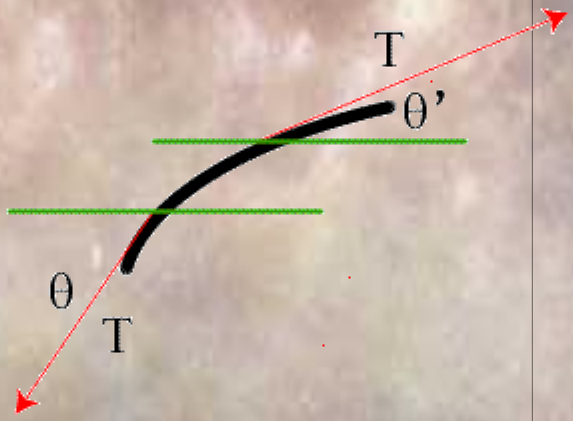
$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \cdot B = 0$$

$$\nabla \times B = \mu_o \varepsilon_o \frac{\partial E}{\partial t}$$

The Wave Equation

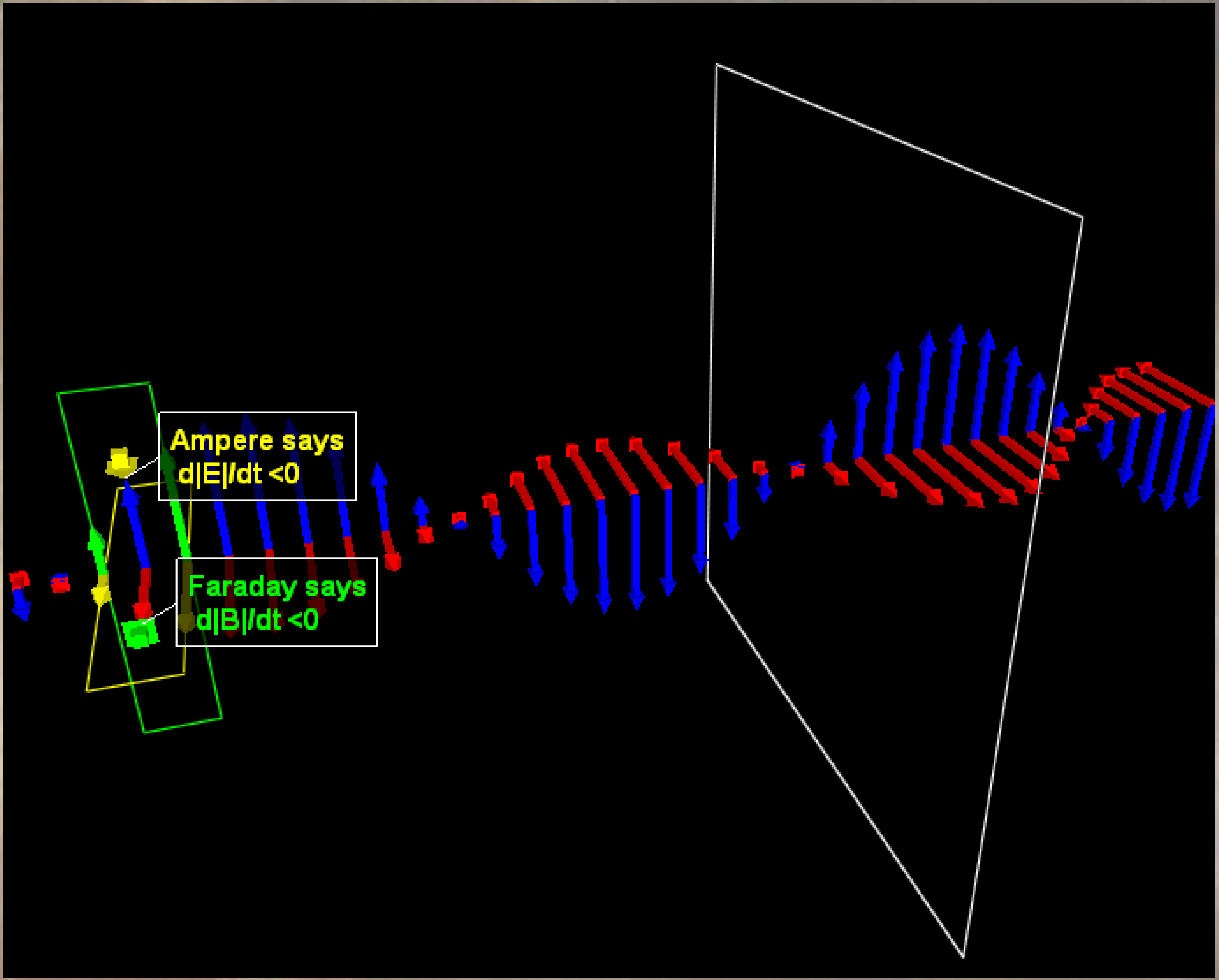
- How fast will a wave travel along a string of density ρ?



Two Ways to M'Es…

- Abstract: $$\nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A$$
- Physical:
  - Imagine a plane wave of electric field in z-direction

$$E(x,t) = \begin{bmatrix} 0 \\ 0 \\ E_z(x,t) \end{bmatrix}$$



Go to Rob Salgado's sim of this

Moving Fields…

- Moving E-Field leads to…

- Moving B-Field leads to…

It's Alive!

- Well, at least it's a wave! Combining the last two equations leads us to:

$$\frac{\partial^2 E_y}{\partial x^2} = \mu_o \varepsilon_o \frac{\partial^2 E_y}{\partial t^2}$$

- example - consider the electric field part of an electromagnetic wave described by:

$$E(x,t) = E_o \sin(kx - \omega t)\, j + E_o \cos(kx - \omega t) k$$

The Poynting Vector

- Light waves (and all electromagnetic waves) carry energy

$$u = u_E + u_B = \frac{EB}{\mu_o c}$$

- A wave has an intensity

$$I = u_{average} c$$

Poynting Vector

$$I = \frac{1}{2}\frac{E_o B_o}{\mu_o} = \left|\vec{S}\right|_{av}$$

$$\vec{S} = \frac{1}{\mu_o}\vec{E}\times\vec{B}$$

Radiation Pressure

- Light waves (and all electromagnetic waves) exert pressure

$$P = \frac{I}{c}$$

Accelerating Charges Radiate Power!

- We can show (by dimensional argument) that an accelerating charge should radiate energy at a rate given by:

$$P = \alpha \frac{kq^2a^2}{c^3}$$

- More detailed argumentation reveals that α = 2/3